Riviera International Academy

Assignment-2077

(Ashad 11, 2077, Thursday)

Class: Ten

Subject-Opt. Mathematics

Source: Photos of exercise are given below.

Work: Read & write all the definitions with all examples.

Do your work neatly.

3. COMPOSITE FUNCTIONS

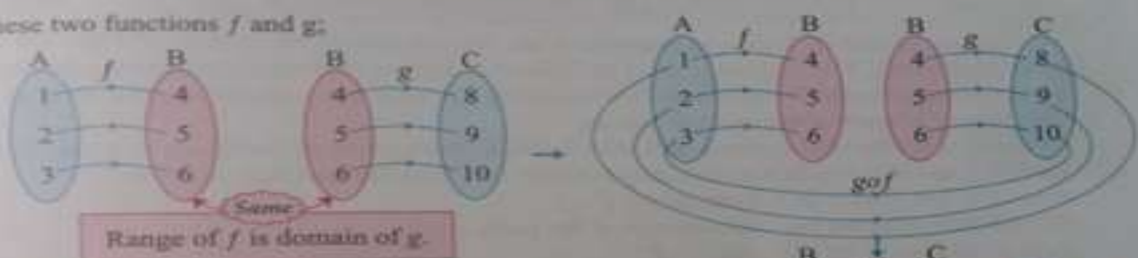
Learning Objectives

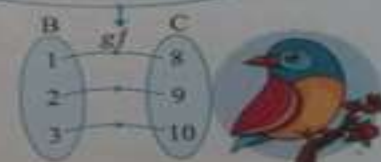
After successful completion of this chapter, the reader should be able to learn and appreciate:

- Introduction to the composite function
- Combination of functions and composite function
- Composite function & Mapping diagram
- Composite function & Algebraic functions

3.1 Introduction

In these two functions f and g:





Function f is defined from set A to set B.
Function g is defined from set B to set C.
The set B is range of f and the domain of g.
In the diagram, the function defined from set A to set C is called the composite function of f and g.

Thus, *if A, B and C are three non empty sets and $f: A \to B$ and $g: B \to C$ be the functions then the composite function of f and g denoted by $g \circ f$ or gf is a function defined from A to C. Mathematically it is defined as: $(g \circ f): A \to C$, $(g \circ f)(x) = g[f(x)]$ for all x of A.*

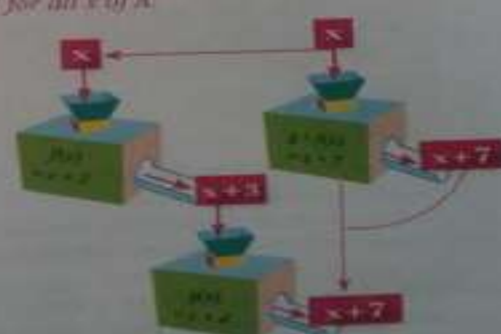
From the diagram; $f(x) = x + 3$ and $g(x) = x + 4$

So, $gf = g(f(x))$
$= g(x + 3)$
$= (x + 3) + 4$

$\therefore gf = x + 7$

Now, When $x = 1$ then $gf = 1 + 7 = 8$
When $x = 2$ then $gf = 2 + 7 = 9$
When $x = 3$ then $gf = 3 + 7 = 10$

Thus, $gf = x + 7$ gives the elements from A to C.



3.2 Properties of Composite Functions

Look at the following diagrams :

(i) $g \circ f(x) = g(f(x))$ (ii) $h \circ (g \circ f)(x) = h[g(f(x))]$ (iii) $[(h \circ g) \circ f](x) = h[g(f(x))]$

3.3 Combination of functions and Composite Functions

Combinations of Functions

The sum, difference, product, or quotient of functions can be found easily.

Sum	$(f+g)(x) = f(x) + g(x)$
Difference	$(f-g)(x) = f(x) - g(x)$
Product	$(f \times g)(x) = f(x) \times g(x)$
Quotient	$(f/g)(x) = f(x)/g(x)$, **as long as $g(x)$ isn't zero.**

The domain of each of these combinations is the intersection of the domain of f and the domain of g. In other words, both functions must be defined at a point for the combination to be defined. One additional requirement for the division of functions is that the denominator can't be zero, but we knew that because it's part of the implied domain.

Basically what the above says is that to evaluate a combination of functions, you may combine the functions and then evaluate or you may evaluate each function and then combine.

Example: Let $f(x) = 5x + 2$ and $g(x) = x^2 - 1$.

We will then evaluate each combination at the point $x = 4$.

$f(4) = 5(4) + 2 = 22$ and $g(4) = 4^2 - 1 = 15$

Expression	Combine, then evaluate			Evaluate, then combine	
$(f+g)(x)$	$= (5x+2)+(x^2-1)$ $= x^2+5x+1$	$(f+g)(4)$	$= 4^2+5(4)+1$ $= 16+20+1$ $= 37$	$f(4)+g(4)$	$= 22+15$ $= 37$
$(f-g)(x)$	$= (5x+2)-(x^2-1)$ $= -x^2+5x+3$	$(f-g)(4)$	$= -4^2+5(4)+3$ $= -16+20+3$ $= 7$	$f(4)-g(4)$	$= 22-15$ $= 7$
$(f\cdot g)(x)$	$= (5x+2)\cdot(x^2-1)$ $= 5x^3+2x^2-5x-2$	$(f\cdot g)(4)$	$= 5(4^3)+2(4^2)-5(4)-2$ $= 5(64)+2(16)-20-2$ $= 330$	$f(4)\cdot g(4)$	$= 22(15)$ $= 330$
$\left(\frac{f}{g}\right)(4)$	$= \frac{5x+2}{x^2-1}$	$\left(\frac{f}{g}\right)(4)$	$= \frac{5(4)+2}{4^2-1} = \frac{22}{15}$	$\frac{f(4)}{g(4)}$	$= \frac{22}{15}$

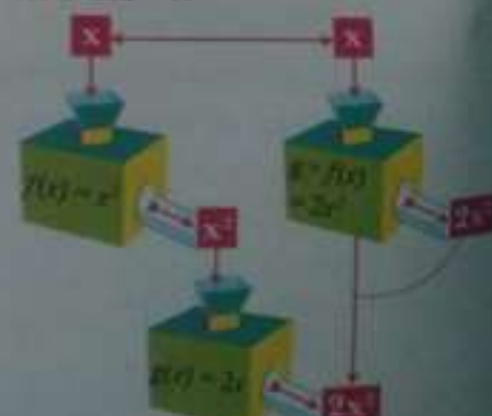
As you can see from the examples, it doesn't matter if you combine and then evaluate or if you evaluate and then combine.

In each of the above problems, the domain is all real numbers with the exception of the division. The domain in the division combination is all real numbers except for 1 and – 1.

Composition of Functions

While the combinations of functions are straight forward and fairly easy, there is another type of combination called composition.

A composition of functions is the applying of one function to another function. The symbol of composition of functions is a small circle (∘) between the function names to represent composition of functions.



$(f \circ g)(x) = f[\,g(x)\,]$ $(g \circ f)(x) = g[\,f(x)\,]$

composed with g of x" and "g composed with f of x"

The function on the outside is always written first with the functions that follow being on the inside. The order is important. Composition of functions is not commutative.

Examples

Let's look at a few examples.

1. $f(x) = 5x + 2$ **and** $g(x) = x^2 - 1$

$$\begin{aligned}(f \circ g)(x) &= f[\,g(x)\,] \\ &= f[\,x^2 - 1\,] \\ &= 5(x^2 - 1) + 2 \\ &= 5x^2 - 5 + 2 \\ &= 5x^2 - 3\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g[\,f(x)\,] \\ &= g[\,5x + 2\,] \\ &= (5x + 2)^2 - 1 \\ &= 25x^2 + 20x + 4 - 1 \\ &= 25x^2 + 20x + 3\end{aligned}$$

2. $f(x) = \sqrt{x}$ **and** $g(x) = 4x^2$

$$\begin{aligned}(f \circ g)(x) &= f[\,g(x)\,] \\ &= f[\,4x^2\,] \\ &= \sqrt{4x^2} \\ &= 2\,|\,x\,|\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g[\,f(x)\,] \\ &= g[\,\sqrt{x}\,] \\ &= 4(\sqrt{x}\,)^2 \\ &= 4x,\ x \geq 0\end{aligned}$$

The square root of (x^2) is the absolute value of x. The square of (the square root of x) is x, but this assumes that x is not negative because you couldn't find the square root of x in the first place if it was (-) ve.

4 Finding Domains on Composition of Functions

Let us consider $(f \circ g)(x)$. We see that g is evaluated at x, so x has to be in the domain of g.

We also see that f is evaluated at $g(x)$, so $g(x)$ has to be in the domain of f.

For $(f \circ g)(x)$,

x is a value that can be plugged into g and gives you a value $g(x)$ that can be plugged into f to get $f(g(x))$.

For $(g \circ f)(x)$,

x is a value that can be plugged into f and gives you a value $f(x)$ that can be plugged into g to get $g(f(x))$.

Let's consider another example again.

Function	Domain	Range
$f(x) = \sqrt{x - 4}$	$x \geq 4$	$y \geq 0$
$g(x) = 1 - x^2$	all reals	$y \leq 1$

When you find $(f \circ g)(x)$, there are two things that must be satisfied:

1. x must be in the domain of g, which means x is a real number.
2. $g(x)$ must be in the domain of f, which means that $(1 - x^2)^2 \geq 4$.
   (when you try to solve this, you get the empty set)

When you find $(g \circ f)(x)$, there are two things that must be satisfied:

1. x must be in the domain of f, which means that $x \geq 4$.
2. $f(x)$ must be in the domain of g which means that the $\sqrt{x - 4}$ must be a real number.
   (that occurs when $x \geq 4$, which we already have stated from the first part)

When you combine the two domains to see what they have in common, you find the intersection to be $x \geq 4$, so that is the where the composition is defined.

3.5 Decomposition of Functions

Decomposition of functions is the reverse of composition of functions. Instead of composing two functions to get a new function, you're breaking apart a composed function into its separate components. There is often more than one way to decompose a function, so your answers may vary from the books. Basically, you want to look at the function and look for an "outside function" and an "inside function". Another thing to look for is repeated patterns and make that the inside function. The outside function is summarized as "the big picture" and the inside function is "what you are doing the big picture to".

Examples

Write each function h as the composition of two functions f and g such that $h(x) = (f \circ g)(x)$

$h(x) = (f \circ g)(x)$	Outside $f(x)$	Inside $g(x)$	Notes
$(1-x)^3$	x^3	$1-x$	The big thing going on is cubing something, so the outside function is a cubing function. $(1-x)$ is what you're cubing, so it's the inside function.
$\sqrt{9-x}$	$\sqrt{x}$	$9-x$	The big thing going on is taking the square root (outside), $(9-x)$ is what you're taking the square root of (inside).
$\frac{4}{(5x^2+2)^2}$	$\frac{4}{x^2}$	$5x^2+2$	Looks like 4 over something squared.
	$4/x$	$(5x^2+2)^2$	An alternative, but correct answer.
$(x+2)^2+2(x+2)+1$	x^2+2x+1	$x+2$	$x+2$ is repeated, so that's a good choice for the inside function. Replace every occurrence of the pattern by x for the outside function.

Example: Find the functions f and g such that $f \circ g = h(x) = (x^2+1)^{50}$.

Solution Here, $h(x) = (x^2+1)^{50}$

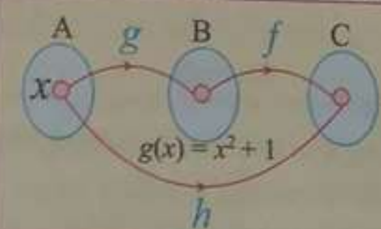
The function h takes x^2+1 and rises it to the power 50.

A natural way to decompose h is to rise the function $g(x) = x^2+1$ to the power 50.

If we let $f(x) = x^{50}$ and $g(x) = x^2+1$

Then, $(f \circ g)(x) = f(g(x))$

$$= f(x^2+1)$$
$$= (x^2+1)^{50}$$
$$= h(x)$$



Other functions f and g may be found for which $fog = h$.

For Example, if $f(x) = x^2$ and $g(x) = (x^2+1)^{25}$ then,

$$(f \circ g)(x) = f(g(x))$$
$$= f((x^2+1)^{25})$$
$$= [(x^2+1)^{25}]^2$$
$$= (x^2+1)^{50} = h(x)$$

Although the functions f and g found as a solution are not unique, there is usually a "natural" selection for f and g that comes to mind first.

Subject- HPE

1. What do you mean by regional development? Explain with its importance.
2. What is urbanization? How is urbanization growing in the context of Nepal?

Subject- Science

1. What is the change in the pressure at the bottom of a drum filled with water if it is brought to Terai from Himalaya? Write with reason.
2. Name the two forces and write the directions applied on a body when it is placed in a liquid.
3. If one of the two boats of each of equal mass is immersed in the sea and another in the Fewatal so that more part of the boat is found immersed in Fewatal, why? In which boat is the more up thrust acted upon and why?
4. State law of floatation and mention one application based on this law.

Subject- Computer

1) Write a program to input base and height of a triangle and calculate its area using SUB procedure. [Hint: a=1/2×base×height]
2) Write a program to input initial velocity, time taken and acceleration of a running car and calculate the distance travelled by it using SUB procedure. [Hint: $s = ut+1/2at^2$]
3) Write a program using FUNCTION......... END FUNCTION to input radius of a circle and print the area and perimeter of a circle.

The End.