Riviera International Academy

Assignment-2077

(Ashwin 09, 2077, Friday)

Class: Ten

Subject- Opt. Mathematics

Source: Photos of exercise are given below.
Work: Read & write all the formulae and practice then twice.
Do your work neatly

1. MULTIPLE ANGLES

Learning Objectives

After successful completion of this chapter, the reader should be able to learn and appreciate:

- To convert trigonometric ratios of 2A in terms of A.
- To convert trigonometric ratios of 3A in terms of A.

1.1 Introduction

In this chapter, we are going to express the trigonometric ratios of multiple angles in terms of trigonometric ratios of angle A.

If A be any angle, then 2A, 3A, 4A, ------ etc are called the multiple angles of A. The angles 2A, 3A, 4A,....... etc can be derived from the results of the previous chapter by putting B = A, 2A etc. in the formulae involving (A + B), etc.

1.2 Ratios of 2A

1. sin 2A

We have, $\sin(A+B) = \sin A.\cos B + \cos A.\sin B$

Putting B = A then,

$\sin(A+B) = \sin(A+A)$

$= \sin A.\cos A + \cos A.\sin A$

$= 2\sin A.\cos A$

Thus, $\sin 2A = 2\sin A.\cos A$

2. cos 2A

$\cos 2A = \cos(A+A)$

$= \cos A.\cos A - \sin A.\sin A$

$= \cos^2 A - \sin^2 A$

Thus, $\cos 2A = \cos^2 A - \sin^2 A$

Again, $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - \sin^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

Thus, $\cos 2A = 1 - 2\sin^2 A$

Similarly,

$\cos 2A = \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A)$

$= 2\cos^2 A - 1$

Thus, $\cos 2A = 2\cos^2 A - 1$

3. tan 2A

$\tan 2A = \tan(A+A)$

$= \dfrac{\tan A + \tan A}{1 - \tan A.\tan A}$

$= \dfrac{2\tan A}{1 - \tan^2 A}$

Thus, $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

4. Carnot formulae

$1 + \cos 2A = 1 + \cos^2 A - \sin^2 A$

$= 2\cos^2 A$

$1 - \cos 2A = 1 - \cos^2 A + \sin^2 A$

$= 2\sin^2 A$

Thus, $1 + \cos 2A = 2\cos^2 A$

Thus, $1 - \cos 2A = 2\sin^2 A$

1.3 Sin 2A and cos 2A in terms of tan A.

1. $\sin 2A = \dfrac{2\sin A\cos A}{1}$

$= \dfrac{2\sin A\cos A}{\cos^2 A + \sin^2 A}$

Dividing numerator and denominator by $\cos^2 A$.

$$= \dfrac{\dfrac{2\sin A\cos A}{\cos^2 A}}{\dfrac{\cos^2 A}{\cos^2 A} + \dfrac{\sin^2 A}{\cos^2 A}}$$

$$= \dfrac{2\tan A}{1 + \tan^2 A}$$

Thus, $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

2. $\cos 2A = \dfrac{\cos^2 A - \sin^2 A}{1}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$

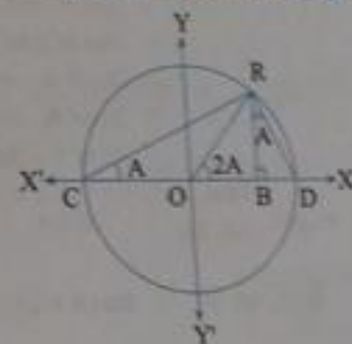
Dividing numerator and denominator by $\cos^2 A$.

$$= \dfrac{\dfrac{\cos^2 A}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\cos^2 A} + \dfrac{\sin^2 A}{\cos^2 A}}$$

$$= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$$

Thus, $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

1.4 The formulae for 2A you have established here may be proved geometrically by using a unit circle as follows:



Draw a unit circle as shown in the figure.

Take a point R and join RO so that $\measuredangle ROX = 2A$.

Join RC, RD and draw RB perpendicular to OX.

Here, OC = OR = OD = 1 unit (radius of circle).

CD is diameter so, $\measuredangle CRD = 90^\circ$.

In ΔRCO, OC = OR

so, $\measuredangle OCR = \measuredangle ORC$ and $\measuredangle OCR + \measuredangle ORC = \measuredangle ROX$.

Hence, $\measuredangle OCR = \measuredangle ORC = A$

Now,

(i) $\sin 2A = \dfrac{RB}{OR} = \dfrac{2RB}{2OR} = \dfrac{2RB}{CD} = 2\cdot\dfrac{RB}{CR}\cdot\dfrac{CR}{CD} = 2\sin A\,.\cos A$

(ii) $\cos 2A = \dfrac{OB}{OR} = \dfrac{2OB}{2OR} = \dfrac{2OB}{CD} = \dfrac{(CO + OB) - (CO - OB)}{CD}$

$= \dfrac{(CO + OB) - (DO - OB)}{CD} = \dfrac{CB - DB}{CD} = \dfrac{CB}{CD} - \dfrac{DB}{CD} = \dfrac{CB}{CR}\cdot\dfrac{CR}{CD} - \dfrac{DB}{RD}\cdot\dfrac{RD}{CD}$ [$\because \measuredangle BRD = A$]

$= \cos A.\cos A - \sin A\,.\sin A$

$\cos 2A = \cos^2 A - \sin^2 A$

(iii) $\tan 2A = \dfrac{RB}{OB} = \dfrac{2RB}{2OB} = \dfrac{2RB}{(CO + OB) - (CO - OB)} = \dfrac{2RB}{(CO + OB) - (DO - OB)} = \dfrac{2RB}{CB - DB}$

$$= \dfrac{\dfrac{2RB}{CB}}{\dfrac{CB}{CB} - \dfrac{DB}{CB}}$$ [$\because$ Dividing by CB]

$$= \dfrac{\dfrac{2RB}{CB}}{1 - \dfrac{DB}{RB}\cdot\dfrac{RB}{CB}} = \dfrac{2\tan A}{1 - \tan^2 A}$$

1.5 Ratios of 3A

1. **sin 3A**

We have, $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin 2A = 2\sin A \cdot \cos A$, $\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$ and $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ etc.

Putting B = 2A then,

$$
\begin{aligned}
\sin 3A &= \sin(A+2A) \\
&= \sin A \cdot \cos 2A + \cos A \cdot \sin 2A \\
&= \sin A \cdot (1 - 2\sin^2 A) + \cos A \cdot 2\sin A \cos A \\
&= \sin A - 2\sin^3 A + 2\sin A(1-\sin^2 A) \\
&= \sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A
\end{aligned}
$$

$\therefore \quad \sin 3A = 3\sin A - 4\sin^3 A$

Thus, *$\sin 3A = 3\sin A - 4\sin^3 A$*

2. **cos 3A**

$$
\begin{aligned}
\cos 3A &= \cos(A+2A) \\
&= \cos A \cdot \cos 2A - \sin A \cdot \sin 2A \\
&= \cos A(2\cos^2 A - 1) - \sin A \cdot 2\sin A \cdot \cos A \\
&= 2\cos^3 A - \cos A - 2\cos A(1-\cos^2 A) \\
&= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A \\
&= 4\cos^3 A - 3\cos A
\end{aligned}
$$

Thus, *$\cos 3A = 4\cos^3 A - 3\cos A$*

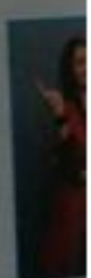
3. **tan 3A**

$$
\tan 3A = \tan(A+2A) = \frac{\tan A + \tan 2A}{1 - \tan A.\tan 2A} = \frac{\tan A + \frac{2\tan A}{1-\tan^2 A}}{1 - \tan A \cdot \frac{2\tan A}{1-\tan^2 A}}
$$

$$
= \frac{\tan A - \tan^3 A + 2\tan A}{1 - \tan^2 A - 2\tan^2 A}
$$

$$
= \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}
$$

Thus, *$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$*



SOLVED EXAMPLES

Example 1 **Prove that:** $\frac{\tan A - \cot A}{\tan A + \cot A} = -\cos 2A$

Solution: Here, LHS

$$
= \frac{\tan A - \cot A}{\tan A + \cot A} = \frac{\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{\frac{\sin^2 A - \cos^2 A}{\cos A \cdot \sin A}}{\frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A}} = \frac{\sin^2 A - \cos^2 A}{\sin^2 A + \cos^2 A}
$$

$$
= \frac{-(\cos^2 A - \sin^2 A)}{1}
$$

$$
= -\cos 2A
$$

$$
= \text{RHS}
$$

Prove that: $\cot 10° - \sqrt{3} = 4\cos 10°$

Here, LHS $= \cot 10° - \sqrt{3}$

$= \dfrac{\cos 10°}{\sin 10°} - \sqrt{3}$

$= \dfrac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°}$

Putting $\sqrt{3} = \tan 60°$, we get

$= \dfrac{\cos 10° - \tan 60° . \sin 10°}{\sin 10°}$

$= \dfrac{\cos 60° . \cos 10° - \sin 60° . \sin 10°}{\cos 60° . \sin 10°}$

$= \dfrac{\cos(60° + 10°)}{\frac{1}{2} . \sin 10°}$

$= \dfrac{2\cos 70°}{\sin 10°}$

$= \dfrac{2\cos(90° - 20°)}{\sin 10°}$

$= \dfrac{2\sin 20°}{\sin 10°}$

$= \dfrac{4\sin 10° . \cos 10°}{\sin 10°}$

$= 4\cos 10°$

= RHS *Proved.*

ample 3 **Prove that: $\sin^4 A = \frac{1}{8}(3 - 4\cos 2A + \cos 4A)$**

lution Here, LHS $= \sin^4 A = (\sin^2 A)^2$

$= \left[\frac{1}{2}(1 - \cos 2A)\right]^2$

$= \frac{1}{4}(1 - 2\cos 2A + \cos^2 2A)$

$= \frac{1}{4}\left\{1 - 2\cos 2A + \left(\frac{1 + \cos 4A}{2}\right)\right\}$

$= \frac{1}{4}\left(\frac{2 - 4\cos 2A + 1 + \cos 4A}{2}\right)$

$= \frac{1}{8}(3 - 4\cos 2A + \cos 4A)$

= RHS *Proved.*

Alternative Method

RHS $= \frac{1}{8}(3 - 4\cos 2A + \cos 4A)$

$= \frac{1}{8}(3 - 4\cos 2A + 2\cos^2 2A - 1)$

$= \frac{1}{8}[2 - 4(1 - 2\sin^2 A) + 2(1 - 2\sin^2 A)^2]$

$= \frac{1}{8} \times 2[1 - 2(1 - 2\sin^2 A) + 1 - 4\sin^2 A + 4\sin^4 A]$

$= \frac{1}{4}[1 - 2 + 4\sin^2 A + 1 - 4\sin^2 A + 4\sin^4 A]$

$= \frac{1}{4} \times 4\sin^4 A$

$= \sin^4 A$

= LHS *Proved.*

ample 4 **Prove that : $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$**

lution Here,

LHS $= \cos 5A$

$= \cos(2A + 3A)$

$= \cos 2A . \cos 3A - \sin 2A . \sin 3A$

$= (2\cos^2 A - 1)(4\cos^3 A - 3\cos A) - 2\sin A . \cos A(3\sin A - 4\sin^3 A)$

$= 8\cos^5 A - 6\cos^3 A - 4\cos^3 A + 3\cos A - 6\sin^2 A . \cos A + 8\sin^4 A . \cos A$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 6(1 - \cos^2 A) . \cos A + 8(1 - \cos^2 A)^2 \cos A$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 6\cos A + 6\cos^3 A + 8\cos A(1 - 2\cos^2 A + \cos^4 A)$

$= 8\cos^5 A - 10\cos^3 A + 3\cos A - 6\cos A + 6\cos^3 A + 8\cos A - 16\cos^3 A + 8\cos^5 A$

$= 8\cos^5 A - 4\cos^3 A - 3\cos A + 8\cos A - 16\cos^3 A + 8\cos^5 A$

$= 16\cos^5 A - 20\cos^3 A + 5\cos A$ = RHS *Proved.*

Example 5 **Prove that:** $\frac{1 + \sin 2A + \cos 2A}{1 + \sin 2A - \cos 2A} = \cot A$

Solution Here,

$$\text{LHS} = \frac{1 + \sin 2A + \cos 2A}{1 + \sin 2A - \cos 2A}$$

$$= \frac{1 + 2\sin A \,.\, \cos A + 2\cos^2 A - 1}{1 + 2\sin A \,.\, \cos A - 1 + 2\sin^2 A}$$

$$= \frac{2\cos A(\sin A + \cos A)}{2\sin A(\cos A + \sin A)}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

$= \text{RHS}$ *Proved.*

Example 6 **Prove that:** $\tan A - \cot A = -2\cot 2A$

Solution Here, $\text{LHS} = \tan A - \cot A$

$$= \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A \,.\, \cos A}$$

$$= \frac{-(\cos^2 A - \sin^2 A) \times 2}{2\sin A \,.\, \cos A}$$

$$= \frac{-2\cos 2A}{\sin 2A}$$

$$= -2\cot 2A$$

$= \text{RHS}$ *Proved.*

Example 7 **If** $\sin A = \frac{1}{2}\left(a + \frac{1}{a}\right)$, **show that** $\sin 3A = -\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$.

Solution Here, $\sin A = \frac{1}{2}\left(a + \frac{1}{a}\right)$

Now, $\sin 3A = 3\sin A - 4\sin^3 A$

$$= \sin A\,(3 - 4\sin^2 A)$$

$$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left\{3 - 4 \times \frac{1}{4}\left(a + \frac{1}{a}\right)^2\right\} = \frac{1}{2}\left(a + \frac{1}{a}\right)\left(3 - a^2 - 2 - \frac{1}{a^2}\right)$$

$$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left(1 - a^2 - \frac{1}{a^2}\right) = -\frac{1}{2}\left(a + \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right) = -\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$$

Thus, $\sin 3A = -\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ *Proved.*

Example 8 **Prove that:** $2\cos 8A + 1 = (2\cos 2A - 1)(2\cos 2A + 1)(2\cos 4A - 1)$

Solution Here, $\text{LHS} = 2\cos 2\,.\,4A + 1$

$$= 2(2\cos^2 4A - 1) + 1$$

$$= 4\cos^2 4A - 2 + 1$$

$$= 4\cos^2 4A - 1$$

$$= (2\cos 4A - 1)(2\cos 4A + 1)$$

$$= (2\cos 4A - 1)\{2(2\cos^2 2A - 1) + 1\}$$

$$= (2\cos 4A - 1)(4\cos^2 2A - 2 + 1)$$

$$= (2\cos 4A - 1)(4\cos^2 2A - 1)$$

$$= (2\cos 4A - 1)(2\cos 2A - 1)(2\cos 2A + 1)$$

$= \text{RHS}$ *Proved.*

Example 9 **Prove that:** $\sqrt{2 + \sqrt{2 + 2\cos 8\theta}} = 2\cos 2\theta$

Solution Here, $\text{LHS} = \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}$

$$= \sqrt{2 + \sqrt{2(1 + \cos 2\,.\,4\theta)}} = \sqrt{2 + \sqrt{2\,.\,2\cos^2 4\theta}} = \sqrt{2 + 2\cos 4\theta}$$

$$= \sqrt{2(1 + \cos 4\theta)} = \sqrt{2\,.\,2\cos^2 2\theta} = 2\cos 2\theta = \text{RHS}$$ *Proved.*

Let us Memorize

Ratios of Multiple Angles

1. $\sin 2A = 2\sin A \cdot \cos A = \dfrac{2\tan A}{1+\tan^2 A}$	3. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$	9. $\cos 3A = 4\cos^3 A - 3\cos A$
2. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$	4. $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$	10. $4\sin^3 A = 3\sin A - \sin 3A$
	5. $2\sin^2 A = 1 - \cos 2A$	11. $4\cos^3 A = \cos 3A + 3\cos A$
	6. $2\cos^2 A = 1 + \cos 2A$	12. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
	7. $\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$	8. $\sin 3A = 3\sin A - 4\sin^3 A$

EXERCISE 1

1. (a) What is multiple angle? (b) Convert cos 2A in terms of cos A.
   (c) Express tan 2A in terms of tan A. (d) What is the relation between cos 2A and sin A?
2. (a) Write the relation between cos 3A and cos A.
   (b) What is the relation between sin 3A and sin A?
   (c) Write the relation between tan 3A and tan A.
3. (a) If $\sin A = \frac{1}{2}$, find the value of cos 2A. (b) If $\cos A = \frac{1}{2}$, find the value of cos 2A.
   (c) If $\tan A = \frac{1}{2}$, find the value of cos 2A. (d) If $\sin A = \frac{1}{2}$, find the value of sin 3A.

SOLVE

1. **If $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, find the value of:**
   (a) sin 2A (b) cos 2B (c) tan 2A (d) sin 3A (e) cos 3B (f) tan 3A
2. **If $\tan A = \frac{3}{4}$, find the value of:**
   (a) tan 2A (b) sin 2A (c) cos 2A (d) tan 3A (e) sin 3A (f) cos 3A
3. **Prove the following identities:**
   (a) $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A} = 2$
   (b) $\dfrac{\sin 5A}{\sin A} - \dfrac{\cos 5A}{\cos A} = 4\cos 2A$
   (c) $\tan 2A + \sin 2A = \dfrac{4\tan A}{1-\tan^4 A}$
   (d) $\dfrac{\cos A}{\cos A - \sin A} - \dfrac{\cos A}{\cos A + \sin A} = \tan 2A$
   (e) $\dfrac{\sin 2A}{\sin A} - \dfrac{\cos 2A}{\cos A} = \sec A$
   (f) $\dfrac{\sin 8A}{\sin 4A} - \dfrac{\cos 8A}{\cos 4A} = \sec 4A$

Prove that:

(a) $\dfrac{1-\sin 2A}{\cos 2A} = \sec 2A - \tan 2A$
(b) $\dfrac{\cos 2A}{1+\sin 2A} = \dfrac{1-\tan A}{1+\tan A}$
(c) $\dfrac{1+\cos 2A}{\sin 2A} = \cot A$
(d) $\dfrac{1+\sec 2A}{\tan 2A} = \cot A$
(e) $\dfrac{\cot A - \tan A}{\cot A + \tan A} = \cos 2A$
(f) $\operatorname{cosec} 2A + \cot 2A = \cot A$
(g) $\dfrac{\sin 2A + \sin A}{1 + \cos A + \cos 2A} = \tan A$
(h) $\dfrac{2\sin 2A + \sin 4A}{2\sin 2A - \sin 4A} = \cot^2 A$
(i) $\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$
(j) $\dfrac{\sin A - \sqrt{1+\sin 2A}}{\cos A - \sqrt{1+\sin 2A}} = \cot A$

(k) $1 + \tan 4A . \tan 2A = \sec 4A$

(l) $\frac{\cos A - \sin A}{\cos A + \sin A} = \sec 2A - \tan 2A$

(m) $\frac{\sin 4A}{\cos 2A} \times \frac{1 - \cos 2A}{1 - \cos 4A} = \tan A$

(n) $\cot 2A + \tan A = \text{cosec } 2A$

(o) $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} = \frac{1}{2}(2 - \sin 2A)$

(p) $\frac{1 + \sin 2A}{1 - \sin 2A} = \left(\frac{1 + \tan A}{1 - \tan A}\right)^2$

(q) $\text{cosec } 2A + \cot 4A = \cot A - \text{cosec } 4A$

(r) $\frac{1}{\tan A} - \frac{1}{\tan 2A} = \frac{1}{\sin 2A}$

5. **Show that:**

(a) $\frac{1 - \tan^2(45° - A)}{1 + \tan^2(45° - A)} = \sin 2A$

(b) $\frac{2 \tan(45° - A)}{1 + \tan^2(45° - A)} = \cos 2A$

(c) $\cot(45° - A) = \tan 2A + \sec 2A$

(d) $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = 4$

(e) $\sqrt{3}\,\text{cosec } 20° - \sec 20° = 4$

(f) $\cos^3 20° + \sin^3 10° = \frac{3}{4}(\cos 20° + \sin 10°)$

(g) $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2 \tan 2\theta$

(h) $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 2 \sec 2\theta$

(i) $\tan\theta + \tan(60° + \theta) + \tan(120° + \theta) = 3 \tan 3\theta$

6. **Prove the followings:**

(a) $\cos^6 A + \sin^6 A = \frac{1}{4}(1 + 3\cos^2 2A)$

(b) $\cos^6 A - \sin^6 A = \cos 2A\left(1 - \frac{1}{4}\sin^2 2A\right)$

(c) $\cos^8 A + \sin^8 A = 1 - \sin^2 2A + \frac{1}{8}\sin^4 2A$

(d) $\cos^3 A . \cos 3A + \sin^3 A . \sin 3A = \cos^3 2A$

(e) $4 \sin A . \cos^3 A - 4 \cos A . \sin^3 A = \sin 4A$

(f) $4 \sin^3 A . \cos 3A + 4 \cos^3 A . \sin 3A = 3 \sin 4A$

(g) $\cos 4A = 8 \cos^4 A - 8 \cos^2 A + 1$

(h) $\sin 5A = 16 \sin^5 A - 20 \sin^3 A + 5 \sin A$

(i) $2 \cos 4A + 1 = (2 \cos A - 1).(2 \cos A + 1).(2 \cos 2A - 1)$

(j) $2 \cos 8A + 1 = (2 \cos A - 1).(2 \cos A + 1).(2 \cos 2A - 1).(2 \cos 4A - 1)$

(k) $2 \cos 16A + 1 = (2 \cos 2A - 1).(2 \cos 2A + 1).(2 \cos 4A - 1).(2 \cos 8A - 1)$

(l) $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2 \cos\theta$

(m) $2 \cos A = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8A}}}$

(n) $\cos^2 A + \sin^2 A \cos 2B = \cos^2 B + \sin^2 B \cos 2A$

(o) $\sin^2 A . \cos 2B - \sin^2 B . \cos 2A = \cos^2 B - \cos^2 A$

(p) $\frac{1}{\tan 3A + \tan A} - \frac{1}{\cot 3A + \cot A} = \cot 4A$

(q) $\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} = \cot 2A$

(r) $\frac{1}{\tan 6A - \tan 2A} - \frac{1}{\cot 6A - \cot 2A} = \cot 4A$

7. (a) If $\tan A = \frac{x}{y}$, prove that: $y.\cos 2A + x.\sin 2A = y$

(b) If $2 \tan A = 3 \tan B$, then prove that: $\tan(A - B) = \frac{\sin 2B}{5 - \cos 2B}$.

8. **If $\cos A = \frac{1}{2}\left(a + \frac{1}{a}\right)$, show that:**

(a) $\cos 2A = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$

(b) $\cos 3A = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

Subject- English

Follow up activity

Subject-Social Studies

Write short notes on Dhan Nach and Chandi Nach.

Subject- Science

1. What is hormone? Write the function of the hormones secreted by thyroid gland.
2. Which gland is called master gland and why?
3. Write the name of the gland which is located on the top of both kidneys. Also write the name of its hormones.
4. What is goitre? How does it occur? Explain.

The End.