

Riviera International Academy

Assignment-2077

(Bhadra 10, 2077, Wednesday)

Class: Ten

Subject- Opt. Mathematics

Source: Photos of exercise are given below

Work: Read & write Remainder theorem do all work from given pages.

Do your work neatly

1. Using remainder theorem, find the remainder if $f(x)$ is divided by $g(x)$ when,

(a) $f(x) = x^2 + 3x + 12$ and $g(x) = x + 1$ (b) $f(x) = x^2 - x^2 + 20$ and $g(x) = x + 2$
(c) $f(x) = 2x^2 - 3x^2 - x + 5$ and $g(x) = x - 4$ (d) $f(x) = 3x^2 - 5x^2 + 2x - 3$ and $g(x) = x - 1$
(e) $f(x) = x^2 - kx^2 - 2x + k + 4$ and $g(x) = x - k$ (f) $f(x) = 2x^2 + 15x + 7$ and $g(x) = 2x + 1$
(g) $f(x) = 2x^2 - 11x^2 + 19x - 10$ and $g(x) = 2x - 5$

2. Let $f(y) = y^3 - ay^2 - 2y + 1$. Find the remainder using synthetic division when $f(y)$ is divided by $y - a$. Also, find $f(a)$. Do you see any relation between the remainder and $f(a)$?

3. $f(x)$ is the polynomial $4x^3 - 12x^2 + 11x - 3$. Use the remainder theorem to find the remainder when $f(x)$ is divided by

(a) $(x - \frac{1}{2})$ (b) $(x + \frac{1}{2})$ (c) $(x - \frac{3}{2})$

4. (a) If a polynomial $2x^3 + 3x^2 + kx + 4$ leaves a remainder $2(4 - k)$ when divided by $(x - 2)$, find the value of k .
(b) If a polynomial $x^3 - ax^2 - 2x + 2a + 6$ leaves a remainder $(a + 2)$ when divided by $(x - a)$, find the value of a .
(c) If both $ax^2 + 2x^2 - 3$ and $x^2 - ax + 4$ leaves the same remainder when divided by $x - 2$, find a .
(d) If both $px^3 - 3x^2 + 9$ and $x^2 - px + 4$ leaves the same remainder when divided by $x + 1$, find p .
(e) A polynomial $4x^4 - (p + 1)x^2 + 8x + 14$ when divided by $2x + 1$ leaves the remainder 8.5. Find the value of p by using remainder theorem.

5. (a) The expression $ax^2 + bx + 1$ has the remainder 2 when it is divided by $(x - 1)$ and when it is divided by $(x + 1)$, the remainder is 4. Find the values of a and b .
(b) The expression $x^2 + bx + c$ has the remainder 1 when it is divided by $(x + 1)$ and when it is divided by $(x - 2)$, the remainder is 13. Find the values of b and c .

8. FACTOR THEOREM

Learning Objectives

After successful completion of this chapter, the reader should be able to learn and appreciate:

- Introduction to Factor theorem
- To find Factor using Factor theorems.

FACTOR THEOREM

Statement: Let $p(x)$ be a polynomial of degree $n > 0$ is divided by a linear polynomial $x - a$ and if remainder $p(a) = 0$ for a real number a , then $(x - a)$ is a factor of $p(x)$.

Proof: We have ; $p(x)$ is a polynomial of degree $n > 0$ and $(x - a)$ is a divisor of $p(x)$ then

$$p(x) = (x - a) \cdot Q(x) + R \text{ where } Q(x) = \text{quotient and } R = \text{remainder.}$$

If $R = 0$ then, $p(x) = (x - a) \cdot Q(x)$.

This shows that $p(x)$ can be expressed in terms of product of $(x - a)$ and $Q(x)$.

Thus, If $R = 0$ then $(x - a)$ is a factor of $p(x)$.



Example
Solution

2. Find the quotient $Q(x)$ and the remainder $R(x)$ from the following polynomials by using synthetic division method.
- (a) When $x^3 + 3x^2 - 8x - 24$ is divided by $x + 3$. (b) When $x^3 - 6x^2 + 11x - 6$ is divided by $x - 2$.
 (c) When $x^4 - 2x^3 - 4x - 12$ is divided by $x + 1$. (d) When $25 - 36x - 40x^2 - 95x^3$ is divided by $x + 1$.
 (e) When $6x^3 + x^2 - 50x + 26$ is divided by $3x - 7$.
3. (a) If $4x^3 - 14x^2 + 2x + 20 = (x - 1)Q(x) + R$. Find the remainder R and polynomial $Q(x)$.
 (b) If $x^4 - 5x^3 + 2x - 5 = (x - 5)Q(x) + R$, find $Q(x)$ and R .
 (c) If $4x^3 - 22x^2 + 38x - 20 = (x - 5)Q(x) + R$, find $Q(x)$ and R .
4. (a) If $f(x) = 2x^2 + 2x^2 - 3x + k$ and $f(2) = 8$, find the value of k .
 (b) If $p(x) = 2x^2 - 3x^2 - 6x + q$ and $p(1) = 0$, find the value of q .
5. (a) Find the value of c for which the polynomial $2x^3 - 7x^2 - x + c$ is exactly divisible by $x + \frac{3}{2}$.
 (b) For what value of c , the polynomial $x^3 + 4x^2 - cx + 8$ is exactly divisible by $x - 2$?
6. Find the polynomials when divisor is $(x + 1)$, quotient $Q(x)$ and remainder $R(x)$ are given below:
 (a) $Q(x) = 2x^2 + 3x + 1$ and $R(x) = -2$ (b) $Q(x) = 12x^2 - 23x + 10$ and $R(x) = 1$
 (c) $Q(x) = 20 - 9x - 20x^2$ and $R(x) = -2$

7.

REMAINDER THEOREM

Learning Objectives

After successful completion of this chapter, the reader should be able to learn and appreciate:

- Introduction to Remainder theorem
- To find remainder using remainder theorem.

Remainder Theorem

Statement: Let $p(x)$ be any polynomial of degree n , $n \geq 1$, and $g(x) = x - a$, where a is any number. If $p(x)$ is divided by $g(x)$, then the remainder is $p(a)$.

Proof: Let us suppose that, when $p(x)$ is divided by $g(x) = (x - a)$, the quotient is $Q(x)$ and remainder is $r(x)$.

So, we have $p(x) = (x - a)Q(x) + r(x)$, where degree of $r(x)$ is less than degree of $(x - a)$.

Since, degree of $(x - a)$ is 1, degree of $r(x)$ is 0.

So, $r(x)$ is a constant, say R .

$$p(x) = (x - a)Q(x) + R$$

$$\text{For } x = a, \quad p(a) = (a - a)Q(a) + R$$

$$= 0 \cdot Q(a) + R$$

$$\therefore p(a) = 0 + R = R$$

This proves the theorem.

Thus, if a polynomial $p(x)$ is divided by a binomial $(x - a)$ then remainder is $p(a)$.

- Note**
- If a polynomial $p(x)$ is divided by a binomial $(x + a)$ then the remainder is $p(-a)$.
 - If a polynomial $p(x)$ is divided by a binomial $(ax + b)$ then the remainder is $p\left(-\frac{b}{a}\right)$.
 - If a polynomial $p(x)$ is divided by a binomial $(ax - b)$ then the remainder is $p\left(\frac{b}{a}\right)$.



Example 1 If $p(x) = 4x^3 - 3x^2 + 2x - 4$, find the remainder when $p(x)$ is divided by

- (a) $x - 4$ (b) $x + 2$ (c) $x + \frac{1}{2}$

Solution

Here,

- (a) Comparing $x - 4$ with $x - a$ then $a = 4$.

By the Remainder Theorem, the required remainder = $p(a)$

$$\begin{aligned} \text{So, } p(a) = p(4) &= 4(4)^3 - 3(4)^2 + 2(4) - 4 \\ &= 256 - 48 + 8 - 4 \end{aligned}$$

$$\therefore p(a) = 212$$

Thus, the remainder is 212.

- (b) Comparing $x + 2$ with $x - a$ then $a = -2$

By the Remainder Theorem, the required remainder = $p(a)$

$$\begin{aligned} \text{So, } p(a) = p(-2) &= 4(-2)^3 - 3(-2)^2 + 2(-2) - 4 \\ &= -32 - 12 - 4 - 4 \end{aligned}$$

$$\therefore p(a) = -52$$

Thus, the remainder is -52 .

- (c) Comparing $x + \frac{1}{2}$ with $x - a$ then $a = -\frac{1}{2}$

By the Remainder Theorem, the required remainder = $p(a)$

$$\begin{aligned} \text{So, Remainder} = p(a) = p\left(-\frac{1}{2}\right) &= 4\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 4 \end{aligned}$$

$$= -\frac{1}{2} - \frac{3}{4} - 1 - 4 = -\frac{25}{4}$$

Thus, the remainder is $-\frac{25}{4}$.

Comparing
 $x + 2 = x - a$
 $\therefore a = -2$



Let us Summarize

- Let $p(x)$ be any polynomial of degree n , $n \geq 1$, and $g(x) = x - a$, where a is any number. If $p(x)$ is divided by $g(x)$, then the remainder is $p(a)$.
- If a polynomial $p(x)$ is divided by a binomial $(x + a)$ then the remainder is $p(-a)$.
- If a polynomial $p(x)$ is divided by a binomial $(ax + b)$ then the remainder is $p\left(-\frac{b}{a}\right)$.
- If a polynomial $p(x)$ is divided by a binomial $(ax - b)$ then the remainder is $p\left(\frac{b}{a}\right)$.



EXERCISE 7

- State Remainder Theorem.
 - If a polynomial $f(x)$ is divided by $(x - a)$, what will be its remainder?
 - If a polynomial $f(x)$ is divided by $(x + b)$, what will be its remainder?
- If a polynomial $q(x)$ is divided by $(ax - b)$, what will be its remainder?
 - If a polynomial $f(x)$ is divided by $(px + q)$, what is its remainder?
- What is the remainder when a polynomial $f(x)$ is divided by $(2x + 3)$?
 - What is the remainder when a polynomial $x^2 - 1$ is divided by $(x - 2)$?
 - What is the remainder when a polynomial $x^2 - 4$ is divided by $(x + 1)$?

LET'S GET READY

Subject- Social Studies

1. Make list of folk musical instructs.

Subject- English

Unit 6 Ex. B Time for grammar ex 1 and 2

Subject- Science

1. Write any five limitations of balanced chemical equation.
2. Write any two differences between endothermic and exothermic reaction.
3. Write the balance chemical equation for the following chemical reactions. Also write the type of the reaction.
 - i. When the mercuric oxide is heated.
 - ii. When hydrochloric acid kept in calcium chloride.
 - iii. When sodium metal kept in water.
 - iv. When sulphuric acid kept in magnesium hydroxide.

The End.