Riviera International Academy

Assignment-2077

(Jestha 22, 2077, Thursday)

Class: Ten

Subject- Opt. Mathematics

Source: Photo of exercise is given below:

1 ALGEBRA

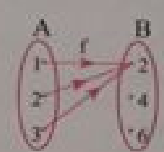
1.1 Relations and Functions

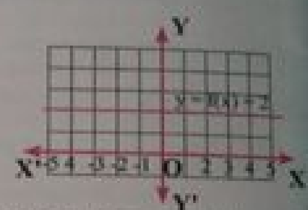
Algebraic Functions

The functions involving algebraic expressions are called algebraic functions. Some examples of algebraic functions are $f(x) = 5$, $f(x) = 2x + 3$, $f(x) = x^2 + 2x + 1$, $f(x) = x^3 + 2$ etc. The algebraic functions have different degrees. Hence, there are different types of algebraic functions. Following are some special types of algebraic functions.

Constant Function

A function $f : A \to B$ is said to be a constant function if, for every $x \in A$, there is same image in B i.e. for all $x \in A$, $y = f(x) = c$, $c \in B$. For example, $f = \{(1, 2), (2, 2), (3, 2)\}$ is a constant function. In this case, the range of f will always be a singleton set.

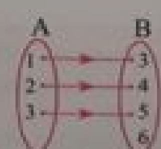


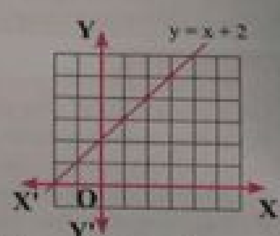


When we discuss the functions over the sets of real numbers, it is difficult to represent them in various ways except with the help of graph. The nature of the graph of function depends on the sets over which we define the functions.

Linear Function

A function $f : A \to B$ is said to be a linear function if for all $x \in A$, $y \in B$, it can be expressed in the form of $y = mx + c$, m and c being constants. For example, $f : A \to B$ is defined as $y = f(x) = x + 2$ where $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$.





In this case, if we draw the graph of the function, all the ordered pairs lie on a straight line.

Identity Function

A function $f : A \to A$ is said to be an identity function if for all $x \in A$, $y = f(x) = x$.

For example, $A = \{1, 2, 3, 4\}$,

$f : A \to A$ is defined as

$f = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

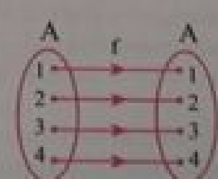
Here, for $x = 1$, $y = f(1) = 1$,

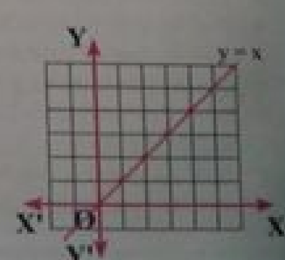
for $x = 2$, $y = f(2) = 2$,

for $x = 3$, $y = f(3) = 3$ and

for $x = 4$, $y = f(4) = 4$.

$\therefore$ f satisfies $y = f(x) = x$, so f is an identity function.





Example 2: *If $f(x) = \dfrac{\sin x}{1 + \cos x}$, find the values of $f(30°)$ and $f\left(\dfrac{\pi}{4}\right)$.*

Solution:

The function given is

$$f(x) = \frac{\sin x}{1 + \cos x}$$

$$f(30°) = \frac{\sin 30°}{1 + \cos 30°}$$

$$= \frac{1/2}{1 + \frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{2 + \sqrt{3}} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

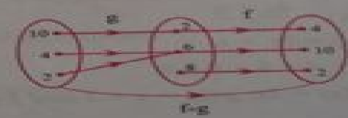
Again, $f\left(\frac{\pi}{4}\right) = \dfrac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}} = \dfrac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2} + 1} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

Example 3: *If f = {(2, 4), (6, 10), (8, 2)} and g = {(4, 6), (10, 2), (2, 6)}, then show that the function f∘g and g∘f in an arrow diagram and find it in ordered pair form.*

Solution:

For f∘g,



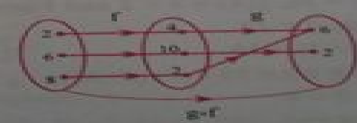
f∘g(4) = f{g(4)} = f(6) = 10

f∘g(10) = f{g(10)} = f(2) = 4

f∘g(2) = f{g(2)} = f(6) = 10

Hence, f∘g = {(4, 10), (10, 4), (2, 10)}

For g∘f,



g∘f(2) = g{f(2)} = g(4) = 6

g∘f(6) = g{f(6)} = g(10) = 2

g∘f(8) = g{f(8)} = g(2) = 6

Hence, g∘f = {(2, 6), (6, 2), (8, 6)}

Example 4: *If f = {(2, 3), (3, 4), (5, 6)} and g∘f = {(2, 6), (3, 10), (5, 17)}, find the function g in the for of set of ordered pairs.*

Solution:

Here, f = {(2, 3), (3, 4), (5, 6)} and g∘f = {(2, 6), (3, 10), (5, 17)}

Showing above information in the mapping diagram

∴ g = {(3, 6), (4, 10), (6, 17)}

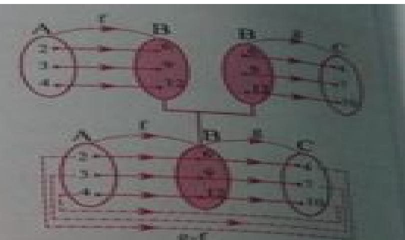
Composite Function

Let A = {2, 3, 4}, B = {6, 9, 12} and C = {4, 7, 10}.

If f : A→B and g : B→C are defined by

f = {(2, 6), (3, 9), (4, 12)} and

g = {(6, 4), (9, 7), (12, 10)}



The general forms of f and g are f(x) = 3x and g(x) = x − 2.

From diagram, it is clear that

f(2) = 6 and g(6) = 4 ∴ g[f(2)] = 4

f(3) = 9 and g(9) = 7 ∴ g[f(3)] = 7

f(4) = 12 and g(12) = 10 ∴ g[f(4)] = 10

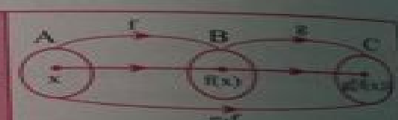
Then we can draw a single function g∘f : A→C as mapping diagram alongside.

g∘f(x) = g[f(x)] = g[3x] = 3x − 2

The function from A to C is called the composite function of f and g.

Definition

Let f : A→B and g : B→C be two functions. Then the new function defined from A to C such that every element of A corresponds with a unique element of C is known as the composite function of f and g. It is denoted by g∘f or simply gf.



Example 1: *Let the function f : R→R be defined by* $f(x) = \begin{cases} 4x^2 - 3 & \text{for } x > 1 \\ 2x + 1 & \text{for } x \le 1 \end{cases}$

find a. $f(2)$ b. $f(0)$ c. $\dfrac{f(h) - f(0)}{h}$ *for* $1 \ge h$

Solution:

a. As 2 > 1, for f(2)

$f(x) = 4x^2 - 3$

or, $f(2) = 4 \times 2^2 - 3 = 4 \times 4 - 3 = 16 - 3 = 13$

b. As 0 < 1, for f(0)

$f(x) = 2x + 1$

or, $f(0) = 2 \times 0 + 1 = 0 + 1 = 1$

c. for, $\dfrac{f(h) - f(0)}{h}$, f(h) = 2h + 1 as $1 \ge h$

or, $\dfrac{f(h) - f(0)}{h} = \dfrac{[2h + 1] - [2 \times 0 + 1]}{h} = \dfrac{2h + 1 - (0 + 1)}{h} = \dfrac{2h + 1 - 1}{h} = \dfrac{2h}{h} = 2$

$(g \circ f \circ h)(2) = g((f \circ h)(2)) = g(f(h(2))) = g(f(2 + 3)) = g(f(5))$
$= g(2 \times 5 - 1) = g(9) = 3 \times 9 = 27$

$(g \circ f \circ h)(2) = 27.$

ercise 1.1A

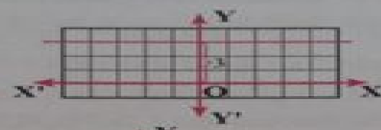
Define the following terms with an example.

a. Inverse function
b. Composite function
c. Identity function
d. Constant function

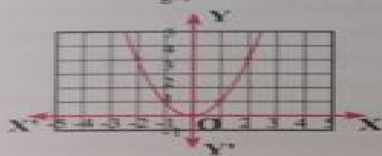
Write the name of the following functions.

a.

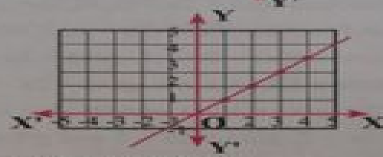
b.



c.



d.



a. A function f is defined on the set of natural number as follows:
$f(x) = \begin{cases} 2x - 7 & \text{for x is even} \\ 4 - 2x & \text{for x is odd} \end{cases}$, find f(2) and f(3).

b. A function f is defined on the set of integers as follows:
$f(x) = \begin{cases} 4x + 1 & \text{for } x < 2 \\ 1 - 3x & \text{for } x \geq 2 \end{cases}$, find f(−3), f(3) and f(8).

a. If $f(x) = \sin 2x + 1$, find the values of $f\left(\frac{\pi^c}{6}\right)$ and $f\left(\frac{\pi^c}{4}\right)$.

b. If $f(x) = \frac{2 \tan x}{1 - \tan^2 x}$, find the values of $f\left(\frac{\pi^c}{6}\right)$ and $f\left(\frac{\pi^c}{2}\right)$.

a. If f = {(−3, 4), (−1, 5), (−2, 6), (2, 4), (0, 3)} and g = {(4, 3), (5, 7), (3, 0), (6, 0)}, find the set of ordered pairs of g∘f and show in a mapping diagram.

b. If f = {(3, 9), (2, 4), (1, 1), (0, 0} and g = {(4, 3), (3, 2), (2, 1), (1, 0)}, then show the function f∘g in an arrow diagram and find it in ordered pair.

c. If f : {(2, 1), (3, 2), (4, 3)} and g(x) is an identity function, then find g∘f and f∘g. Also show them in an arrow diagram.

a. If the functions f and g are given by f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}, find (i) f∘g(2), f∘g(5) and f∘g(1) (ii) g∘f(1), g∘f(3) and g∘f(4)

b. If f = {(2, 3), (4, 6), (−2, 4)} and g = {(4, 5), (3, 7), (6, 8)}, find the value of the following composite functions.
(i) g∘f(2) (ii) g∘f(4) (iii) g∘f(−2)

Example 5: *If $f : R \rightarrow R$ and $g : R \rightarrow R$ be the two functions defined by $f(x) = 3x + 7$ and $g(x) = 2(x - 8)$, find $(g \circ f)(x)$ and $(f \circ g)(x)$ and test whether $(g \circ f)(x) = (f \circ g)(x)$ or not.*

olution:

Here, $f(x) = 3x + 7$ and $g(x) = 2(x - 8)$.

ow, $(g \circ f)(x) = g(f(x))$
$= g(3x + 7)$
$= 2\{(3x + 7) - 8\}$
$= 2(3x - 1)$
$= 6x - 2$
$\therefore (g \circ f)(x) = 6x - 2$

and $(f \circ g)(x) = f(g(x))$
$= f(2(x - 8))$
$= f(2x - 16)$
$= 3(2x - 16) + 7$
$= 6x - 48 + 7$
$= 6x - 41$
$\therefore (f \circ g)(x) = 6x - 41$

ence, $(g \circ f)(x) \neq (f \circ g)(x)$

xample 6: *If $f(x) = 2x - 5$ and $(f \circ g)x = 4x + 3$, then find the equation of linear function g(x) and g(2).*

olution:

$(f \circ g)x = 4x + 3$
or, $f\{g(x)\} = 4x + 3$
or, $2g(x) - 5 = 4x + 3$
or, $2g(x) = 4x + 3 + 5$
or, $2g(x) = 4x + 8$
or, $g(x) = \frac{4x + 8}{2}$
$\therefore$ $g(x) = 2x + 4.$

ow, $g(2) = 2 \times 2 + 4 = 4 + 4 = 8.$

xample 7: *If $f(x) = 3x$, $g(x) = x + 2$ and $f \circ g(x) = 18$, find the value of x.*

olution:

Here, $f(x) = 3x$, $g(x) = x + 2$ and $f \circ g(x) = 18$.
i.e. $f[g(x)] = 18$
or, $f(x + 2) = 18$
or, $3(x + 2) = 18$
or, $x + 2 = 6$
$\therefore$ $x = 4$

xample 8: *If $f(x) = 2x - 1$, $g(x) = 3x$ and $h(x) = x + 3$, find*
a. $(f \circ g \circ h)(x)$ *b. $(g \circ f \circ h)(2)$*

lution:

We have, $f(x) = 2x - 1$, $g(x) = 3x$ and $h(x) = x + 3$.

$(f \circ g \circ h)(x) = f((g \circ h)(x)) = f(g(h(x))) = f(g(x + 3)) = f(3(x + 3))$
$= f(3x + 9) = 2(3x + 9) - 1 = 6x + 17$

$(f \circ g \circ h)(x) = 6x + 17$

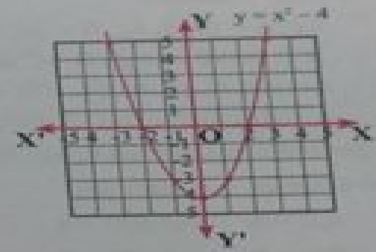
Quadratic Function

A function f : A→B is said to be a quadratic function if, for every x∈A, y∈B, it can be expressed in the form of $y = ax^2 + bx + c$, a ≠ 0, b and c are constants. It represents a curve of conic section named parabola (U shaped).

For example, $y = f(x) = x^2 - 4$ is a quadratic function.

We find the value of f(x) for some values of x and corresponding values of y, then the graph of the function is as shown.

x	0	1	2	−1	−2	3	−3
y	−4	−3	0	−3	0	5	5

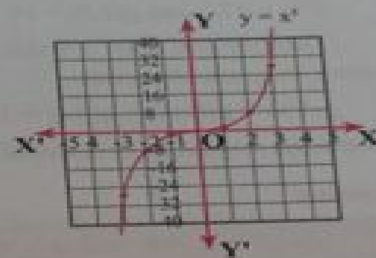


Cubic Function

A function f : A→B is said to be a cubic function if, for all x∈A, y∈B, it can be expressed in the form of $y = ax^3 + bx^2 + cx + d$, a ≠ 0, b, c and d are constants.

For example, $y = x^3$ is a cubic function.

x	0	1	−1	2	−2	3	−3
y	0	1	−1	8	−8	27	−27



Trigonometric Function

A function f : A→B is said to be a trigonometric function if the function f involves trigonometric ratios: sine, cosine, tangent etc. For example, y = sin x, y = cos x, y = sin x + cos x etc. are the trigonometric functions.

Consider a sine function, y = sin x

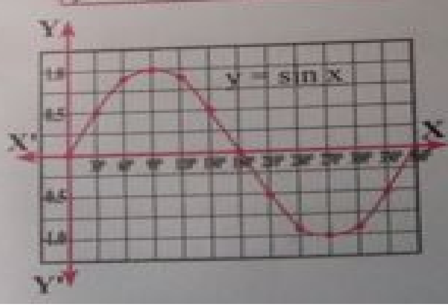
x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y = sin x	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

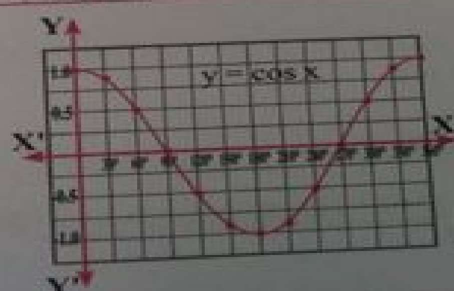
Consider a cosine function, y = cos x

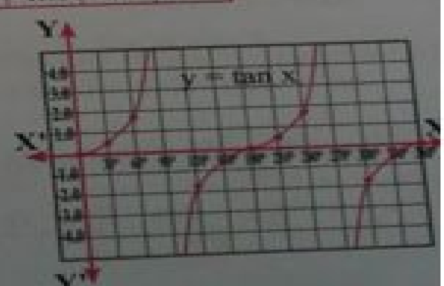
x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y = cos x	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1

Consider a tangent function, y = tan x

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y = tan x	0	0.58	1.73	∞	-1.73	-0.58	0	0.58	1.73	∞	-1.73	-0.58	0







Homework: Do all the examples and complete exercise according to class.

Subject- Mathematics

Source: Textbook of Mathematics-10.

Homework: Do all the exercises of Double Set.

Subject- Social Studies

1. Make a list of the powers distributed to different units, central, provincial and local level.
2. List down negative and positive aspects of federalism.

   Source: Google Classroom

The End.